mmWave Channel Estimation via Approximate Message Passing with Side Information

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Overview

- Increase in mobile user demand makes mmWave frequency band key asset for next-generation cellular networks
- Mobility of users and scattering obstacles means parameters underlying millimeter channels vary dynamically over time
- Dynamic structure can be used as side information

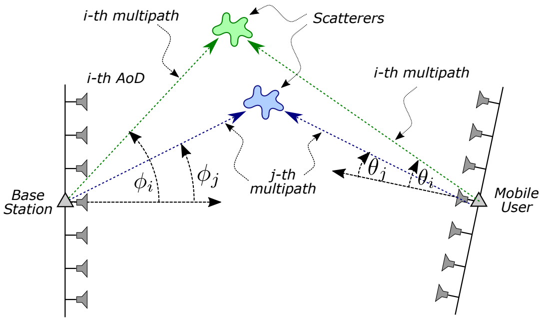
Goal

Channel estimation aided by side information provided by dynamic channel structure

Outcome

Improved estimation quality and reducing training overhead

System Model for mmWave Communication



Representative i-th and j-th multipaths are shown, along with corresponding angles of arrival $\{\theta_i, \theta_j\}$ and departure $\{\phi_i, \phi_j\}$.

mmWave Channel Model

Time-varying channel model with M_t transmit and M_r receive antennas at time k,

$$\mathbf{H}_k = \sqrt{M_\mathsf{t} M_\mathsf{r} \gamma}\, \alpha_k \, \mathbf{a}(\theta_k, M_\mathsf{r}) \, \mathbf{a}(\phi_k, M_\mathsf{t})^{\mathrm{H}},$$

where

γ	signal-to-noise ratio
α_k	complex path gains with $\mathcal{CN}(0,1)$
θ_k & ϕ_k	angles of arrival (AoA) & departure (AoD)

Assuming uniform linear array, array response vectors are

$$\mathbf{a}(\theta_k, M_\mathsf{r}) = \frac{1}{\sqrt{M_\mathsf{r}}} \begin{bmatrix} 1 & e^{j2\pi \frac{d_\mathsf{a}}{\lambda} \sin(\theta_k)} & \cdots & e^{j2\pi \frac{d_\mathsf{a}}{\lambda} (M_\mathsf{r}-1) \sin(\theta_k)} \end{bmatrix}$$

$$\mathbf{a}(\phi_k, M_\mathsf{t}) = \frac{1}{\sqrt{M_\mathsf{t}}} \begin{bmatrix} 1 & e^{j2\pi \frac{d_\mathsf{a}}{\lambda} \sin(\phi_k)} & \cdots & e^{j2\pi \frac{d_\mathsf{a}}{\lambda} (M_\mathsf{t}-1) \sin(\phi_k)} \end{bmatrix},$$

where d_a is antenna spacing, λ wavelength.

Time Variation in Path Gains and AoA & AoD

[Zhang et al '16; Va et al '16; Jayaprakasam et al '16]

Complex Path Gain Over Time

Modeled as first order auto-regressive (AR) process,

$$\alpha_{k+1} = \rho \alpha_k + u_k^{\alpha},$$

where

α_k	path gain
ρ	correlation coefficient
u_k^{α}	innovation noise with $\mathcal{CN}(0, 1-\rho^2)$

Angular Variation Over Time

Modeled as a Gaussian noise process

$$\theta_{k+1} = \theta_k + u_k^{\theta}, \qquad \phi_{k+1} = \phi_k + u_k^{\phi},$$

with u_k^{θ}, u_k^{ϕ}, innovation noise modeled as $\mathcal{CN}(0, \sigma^2)$.

Side Information

Angular Variation and Side Information

- We estimate channel matrix $\mathbf{H}_k$ for each block k
- Have access to $\mathbf{H}_{k-1}$ from previous block, $k-1$
- Use $\mathbf{H}_{k-1}$ as side information (SI) in estimating $\mathbf{H}_k$
- Will see that approximate message passing incorporates SI well

Key Idea

Channel estimation using approximate message passing aided by side information provided by dynamic channel structure

Approximate Message Passing

For channel estimation task, we use class of low-complexity algorithms, referred to as **approximate message passing** or AMP.

[Donoho et al '09; Krzakala et al '12; Montanari '12; Rangan '11]

AMP with Side Information

AMP-SI recently introduced algorithmic framework that incorporates side information (SI) into AMP.

[Ma et al '18; Liu et al '19]

Channel Estimation Model

Signal vector received by user,

$$\mathbf{y}_{k,i} = \mathbf{H}_k \mathbf{s}_{k,i} + \mathbf{n}_{k,i},$$

where

- $\mathbf{n}_{k,i}$ is measurement noise that follows $\mathcal{CN}(\mathbf{0}_{M_\mathrm{r}}, \mathbf{I}_{M_\mathrm{r}})$,
- $\mathbf{s}_{k,i} \in \mathbb{C}^{M_\mathrm{t}}$ unit-energy vector representing pilot symbols transmitted at time i in transmission block k, and
- we assume the received signals are uncorrelated.

In any block k, there are T_p such received signals. Using matrix notation for aggregate receive signal,

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{S}_k + \mathbf{N}_k$$

where

$$\mathbf{Y}_k = [\mathbf{y}_{k,1} | \cdots | \mathbf{y}_{k,T_\mathrm{p}}] \in \mathbb{C}^{M_\mathrm{r} \times T_\mathrm{p}}$$
$$\mathbf{S}_k = [\mathbf{s}_{k,1} | \cdots | \mathbf{s}_{k,T_\mathrm{p}}] \in \mathbb{C}^{M_\mathrm{t} \times T_\mathrm{p}}$$
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In any block k, there are T_p such received signals. Using matrix notation for aggregate receive signal,

$$\mathbf{Y}_k^\mathrm{T} = \mathbf{S}_k^\mathrm{T} \mathbf{H}_k^\mathrm{T} + \mathbf{N}_k^\mathrm{T}$$

where

$$\mathbf{Y}_k = [\mathbf{y}_{k,1} | \cdots | \mathbf{y}_{k,T_\mathrm{p}}] \in \mathbb{C}^{M_\mathrm{r} \times T_\mathrm{p}}$$
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mmWave Channel Estimation via AMP-SI

Initialize with $\widehat{\mathbf{H}}_k^t = \mathbf{0}$ and for $t \geq 0$, compute

$$\text{residual} \quad \mathbf{R}^t = \mathbf{Y}_k^{\mathrm{T}} - \mathbf{S}_k^{\mathrm{T}} \widehat{\mathbf{H}}_k^t + \mathbf{R}^{t-1} \langle \mathit{div}\ \eta_t(\mathbf{V}^t, \mathsf{SI}_{k-1}) \rangle,$$

$$\text{estimate} \quad \widehat{\mathbf{H}}_k^{t+1} = \eta_t(\mathbf{V}^t, \mathsf{SI}_{k-1}),$$

$$\text{pseudo-data} \quad \mathbf{V}^{t+1} = \mathbf{S}_k^* \mathbf{R}^t + \widehat{\mathbf{H}}_k^t,$$

Key Property: $\mathbf{V}^t$ is approximately equal in distribution to $\mathbf{H}_k^{\mathrm{T}}$ plus i.i.d. Gaussian noise with variance

$$\tau_t^2 \approx ||\mathbf{R}^t||^2 / (M_{\mathrm{r}} T_{\mathrm{p}})$$

Estimation quality depends on choice of *denoiser*, denoted $\eta(\cdot, \cdot)$.

Denoisers for AMP-SI

1. **Conditional expectation** denoiser:

$$\eta_t(\mathbf{V}^t, \mathsf{SI}_{k-1}) = \mathbb{E}[\mathbf{H}_k | \mathbf{V}^t = \mathbf{H}_k + \tau_t \mathbf{G}, \mathsf{SI}_{k-1} = \widehat{\mathbf{H}}_{k-1}].$$

   - Has some nice MMSE properties
   - Computationally difficult

2. **Maximum a posteriori** denoiser: compute $(\widehat{\theta}_k, \widehat{\Phi}_k, \widehat{\alpha}_k)$ that maximizes the posterior,

$$f(\theta_k, \Phi_k, \alpha_k | \mathbf{V}^t = \mathbf{H}_k + \tau_t \mathbf{G}, \mathsf{SI}_{k-1} = (\widehat{\theta}_{k-1}, \widehat{\Phi}_{k-1}, \widehat{\alpha}_{k-1})).$$

   - Sub-optimal in terms of MMSE
   - Computational advantages

Numerical Results

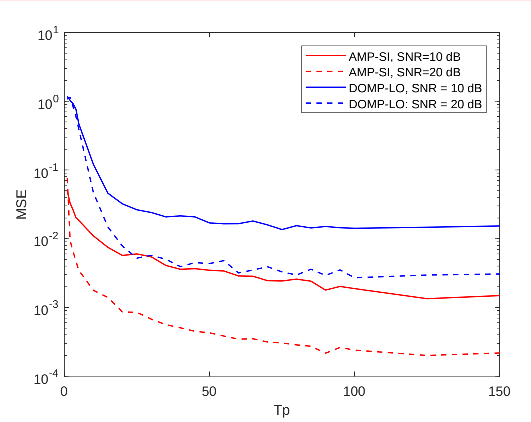

Figure: MSE as function of number of pilots T_{p} per transmission block. AMP-SI used 300 iterations. Our communications setting used $M_{\mathrm{t}} = 16$, $M_{\mathrm{r}} = 8$, $\rho = 0.995$, $\sigma_\theta^2 = \sigma_\phi^2 = 1$, and $\gamma \in \{10, 20\}$ dB, representing a mmWave channel with reasonable time variation.

Summary and Future Work

Summary

- mmWave channel estimation with time-varying parameters and a single path
- channel estimates at time block k are used as side information when estimating the channel at block $k+1$
- employ an SI-aided (complex) AMP algorithm
- compare performance to a benchmark based on orthogonal matching pursuit

Future Work

- multipath extensions
- birth-death-drift dynamics between blocks
- evaluate spectral efficiency of algorithm along with hybrid/digital beamforming schemes as function of training length
- compare performance gains over existing methods